## On the concentration of the chromatic number of random hypergraphs

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The talk deals with estimating the probability threshold for $r$-colorability property in a random hypergraph. Let $H(n,k,p)$ denote the classical binomial model of a random $k$-uniform hypergraph: every edge of a complete $k$-uniform hypergraph on $n$ vertices is included into $H(n,k,p)$ as an edge independently with probability $p \in (0,1)$.

We study the question of estimating the probability threshold for the $r$-colorability property of $H(n,k,p)$. Recall that a hypergraph is $r$-colorable if there exists a vertex coloring with $r$ colors without monochromatic edges. It is well known that for fixed $r \geqslant 2$ and $k \geqslant 2$, this threshold appears in a sparse case when the expected number of edges is a linear function of $n$: $p\binom{n}{k} = cn$ for some fixed $c > 0$.

The following result gives a new lower bound for the $r$-colorability threshold.

**Theorem.** *Let $k \geqslant 4$, $r \geqslant 2$ be integers and $c > 0$. Then there exist absolute constants $C > 0$ and $d_0 > 0$ such that if $\max(r,k) > d_0$ and*

$$c < r^{k-1}\ln r - \frac{\ln r}{2} - \frac{r-1}{r} - C\cdot\frac{k^2\ln r}{r^{k/3-1}}, \tag{1}$$

*then*

$$\mathsf{P}\left(H(n,k,cn/\binom{n}{k}) \text{ is } r\text{-colorable}\right) \to 1 \text{ as } n \to +\infty.$$

Theorem improves the previous result from [1] and provides a bounded gap with the known upper bound. The estimate (1) is only $\frac{r-1}{r} + O\left(\cdot\frac{k^3\ln r}{r^{k/3-1}}\right)$ less than the upper bound from [1]. If the value of the parameter $r$ is much greater than $k$, then a slightly better result is known [2]. The proof is based on a new approach to the second moment method.

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### References

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2. P. Ayre, A. Coja-Oghlan, C. Greenhill, "Hypergraph coloring up to condensation", arXiv:1508.01841 (2015).